

CS 5283: Computer Network Programming: Spring 2004

Homework 4 Solutions: Probability, Random Variables, & Stochastic Processes

1. Problem 7.5 from the text. (pg 180)

- (a) The sample space consists of the 36 equally probable ordered pairs of integers from 1 to 6. The distribution of X is given in the following table.

| | | | | | | |
|-----|------|------|------|------|------|-------|
| xi: | 1 | 2 | 3 | 4 | 5 | 6 |
| pi: | 1/36 | 3/36 | 5/36 | 7/36 | 9/36 | 11/36 |

For example, there are 3 pairs whose maximum is 2: (1, 2), (2, 2), (2, 1).

- (b) Based on the above table, we can calculate $E[X]$ as $E[X] = \sum x_i p_i = 4.5$. Similarly $E[X^2] = 22.0$. Then we have $Var[X] = E[X^2] - E[X]^2 = 1.75$. Std. Dev = $\sqrt{1.75} = 1.3$.

2. Problem 7.6 from the text. (pg 180)

(a)

| | | | | | | |
|-----|-----|-----|-----|-----|-----|-----|
| xi: | -1 | 2 | 3 | -4 | 5 | -6 |
| pi: | 1/6 | 1/6 | 1/6 | 1/6 | 1/6 | 1/6 |

- (b) $E[X] = -1/6$. On average the player loses, so the game is unfair.

3. Problem 7.7 from the text. (pg 180)

(a)

| | | | | |
|-----|---------|--------|--------|-------|
| xi: | -E | E | 2E | 3E |
| pi: | 125/216 | 75/216 | 15/216 | 1/216 |

- (b) $E[X] = -17E/216 = -0.8E$. Here's another game to avoid.

4. Problem 7.8 from the text. (pg 180)

a. $E[X^2] = Var[X] + \mu^2 = 2504$

b. $Var[2X + 3] = 4Var[X] + Var[3] = 16$; standard deviation = 4

c. $Var[-X] = (-1)^2 Var[X] = 4$; standard deviation = 2

5. Problem 7.9 from the text. (pg 180)

The probability density function $f(r)$ must be a constant in the interval (900, 1100) and its integral must equal 1; therefore $f(r) = 1/200$ in the interval (900, 1100). Then $Pr[950 \leq r \leq 1050] = \frac{1}{200} \int_{950}^{1050} dr = 0.5$.

6. Problem 7.12 from the text. (pg 181)

a. No. X and Y are dependent because the value of X determines the value of Y .

$$b. E[XY] = (-1)(1)(0.25) + (0)(0)(0.5) + (1)(1)(0.25) = 0$$

$$E[X] = (-1)(0.25) + (0)(0.5) + (1)(0.25) = 0$$

$$E[Y] = (1)(0.25) + (0)(0.5) + (1)(0.25) = 0.5$$

$$\text{Cov}(X, Y) = E[XY] - E[X]E[Y] = 0$$

c. X and Y are uncorrelated because $\text{Cov}(X, Y) = 0$