Regular Grammars

Chapter 7

Regular Grammars

A regular grammar G is a quadruple (V, Σ, R, S), where:

- V is the rule alphabet, which contains nonterminals and terminals,
- Σ (the set of terminals) is a subset of V,
- R (the set of rules) is a finite set of rules of the form:

$$X \rightarrow Y,$$

- S (the start symbol) is a nonterminal.

Regular Grammars

In a regular grammar, all rules in *R* must:

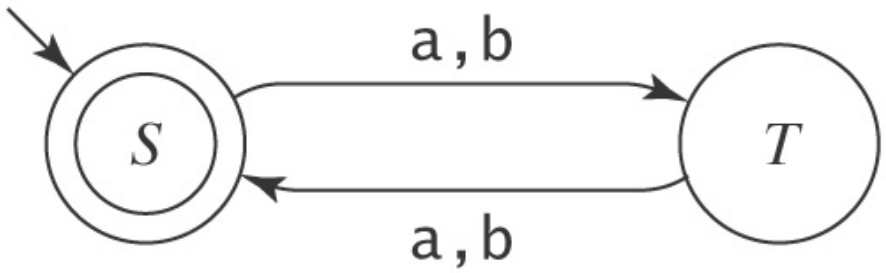
- have a left hand side that is a single nonterminal
- have a right hand side that is:
  - ε, or
  - a single terminal, or
  - a single terminal followed by a single nonterminal.

Legal: $S \rightarrow a$, $S \rightarrow \varepsilon$, and $T \rightarrow aS$

Not legal: $S \rightarrow aSa$ and $aSa \rightarrow T$

Regular Grammar Example

$L = \{w \in \{a, b\}^* : |w| \text{ is even}\}$ $((aa) \cup (ab) \cup (ba) \cup (bb))^*$



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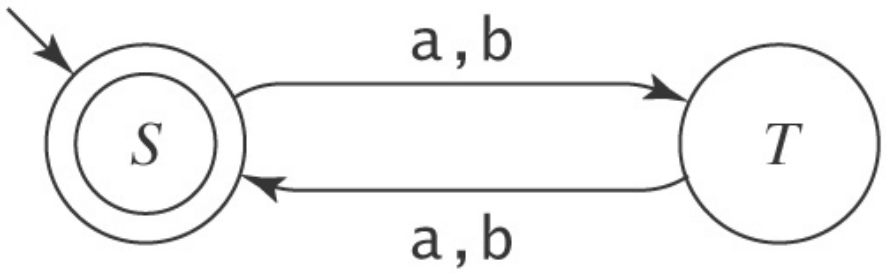


$S \rightarrow \varepsilon$
$S \rightarrow aT$
$S \rightarrow bT$
$T \rightarrow a$
$T \rightarrow b$
$T \rightarrow aS$
$T \rightarrow bS$

Regular Languages and Regular Grammars

Theorem: The class of languages that can be defined with regular grammars is exactly the regular languages.

Proof: By two constructions.

Regular Languages and Regular Grammars

Regular grammar → FSM:

grammartofsm($G = (V, \Sigma, R, S)$) =

1. Create in M a separate state for each nonterminal in V.
2. Start state is the state corresponding to S .
3. If there are any rules in R of the form $X \rightarrow w$, for some $w \in \Sigma$, create a new state labeled #.
4. For each rule of the form $X \rightarrow w\ Y$, add a transition from X to Y labeled w.
5. For each rule of the form $X \rightarrow w$, add a transition from X to # labeled w.
6. For each rule of the form $X \rightarrow \varepsilon$, mark state X as accepting.
7. Mark state # as accepting.

FSM → Regular grammar: Similarly.

Example 1 - Even Length Strings

$S \rightarrow \varepsilon$
$S \rightarrow \mathrm{a}T$
$S \rightarrow \mathrm{b}T$

$T \rightarrow \mathrm{a}$
$T \rightarrow \mathrm{b}$
$T \rightarrow \mathrm{a}S$
$T \rightarrow \mathrm{b}S$

Strings that End with aaaa

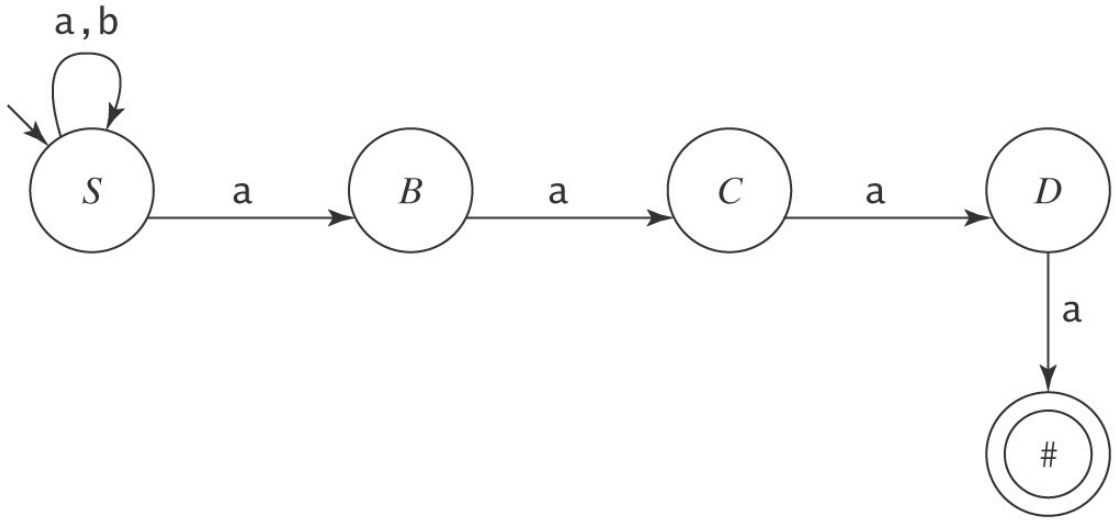
$L = \{w \in \{a, b\}^* : w \text{ ends with the pattern } aaaa\}$.

$$S \to aS$$
$$S \to bS$$
$$S \to aB$$
$$B \to aC$$
$$C \to aD$$
$$D \to a$$

Strings that End with aaaa

$L = \{w \in \{a, b\}^* : w \text{ ends with the pattern } aaaa\}$.

$S \rightarrow aS$
$S \rightarrow bS$
$S \rightarrow aB$
$B \rightarrow aC$
$C \rightarrow aD$
$D \rightarrow a$



Example 2 – One Character Missing

$S \to \varepsilon$
$S \to aB$
$S \to aC$
$S \to bA$
$S \to bC$
$S \to cA$
$S \to cB$

$A \to bA$
$A \to cA$
$A \to \varepsilon$
$B \to aB$
$B \to cB$
$B \to \varepsilon$

$C \to aC$
$C \to bC$
$C \to \varepsilon$