Notes:

- Read Course Information: Section 7 (Miscellaneous) and Section 9 (Academic Dishonesty or Misconduct).
- When you are giving a construction, example, etc., provide a justification with your argument. Your solutions to numerical problems must contain the derivation of your answers. In all of your presentations, strive for correctness, completeness, and clarity. When in doubt about the assumptions of problems, the interpretations of wording, etc., consult the instructor.
- You should strive to complete all problems assigned, and a subset of them will be graded.

1. Read the notes above carefully.

2. Do [Sip12] Do Chapter 1, exercise 1.21 (a). Notes: You may use the procedure presented in lecture. Show all the (important) intermediate work.

3. A finite automaton is called non-crossing if its state-transition diagram can be drawn in the plane without having any edge cross.
   (a) Prove that every regular language is accepted by a non-crossing finite automaton.
   (b) Construct a regular language that is not accepted by any non-crossing deterministic finite automaton.

4. We define a binary relation \( \theta \) on the family of all languages over an alphabet \( \Sigma \): for two languages \( A \) and \( B \) over \( \Sigma \), \( A \theta B \) if and only if (1) \( A \subseteq B \), and (2) \( B \) contains infinitely many strings (over \( \Sigma \)) that are not in \( A \).
   Prove that, if \( A \) and \( B \) are two regular languages over \( \Sigma \) such that \( A \theta B \), then there exists a regular language \( C \) over \( \Sigma \) such that \( A \theta C \) and \( C \theta B \).

5. Consider the following two languages over the alphabet \( \{0, 1\} \):

   \[
   L_1 = \{1^n x \mid x \in \{0, 1\}^* \text{ and } x \text{ contains at least } n \text{ 1s, for } n \geq 1\},
   \]
   \[
   L_2 = \{1^n x \mid x \in \{0, 1\}^* \text{ and } x \text{ contains at most } n \text{ 1s, for } n \geq 1\}
   \]

   Study the regularity of the languages \( L_1 \) and \( L_2 \).

6. Prove that if \( L \) is a regular language over an alphabet \( \Sigma \), then, in first-order statement,

   \[
   \forall y \in \Sigma^* \exists \text{ positive integers } m, n [ (m > n) \land \forall z \in \Sigma^* (y^m z \in L \text{ if and only if } y^n z \in L)].
   \]

   Use the above result to study the regularity of the following languages:
   (a) \( L_1 = \{0^i \mid i > 0\} \).
   (b) \( L_2 = \{uu^i v \mid u, v \in (a+b)^+\} \).