1. The “closed-book/notes” examination contains 3 problems. You have 50 minutes for 25 points.
2. Show all important steps in your work. Your answers will be graded on its correctness and clarity.

1. [5 points] Construct the state-transition diagram of a deterministic finite automaton that accepts the following language:

   \( \{ x \in \{a, b\}^* \mid \#_a(x) \text{ is even, and } \#_b(x) \text{ is divisible by } 3 \} \).

   (Note: \( \#_u(v) \) denotes the number of occurrences of a substring \( u \) in a string \( v \).) Note: A brief and precise interpretation of the states of your machine is required.
2. [10 points] Let $\Sigma$ be the alphabet of 3 symbols: $\Sigma = \{a, b, c\}$.

(a) [5 points] Consider the following language:

$$L_1 = \{ w \in \Sigma^* \mid \text{there exists a symbol of } \Sigma \text{ not appearing in } w \}.$$ 

Give the formal 5-tuple definition and the state-transition diagram of a nondeterministic finite automaton with at most 5-states (with or without $\epsilon$-transitions) that accepts $L_1$. Notes: A brief and precise interpretation of the states of your machine is required, and no credit will be given to finite automaton with more than 5 states.
(b) [5 points] Consider the following language:

\[ L_2 = \{ w \in \Sigma^* \mid w \text{ has a substring of length } 3 \text{ containing each of the symbols of } \Sigma \} \].

Give the state-transition diagram of a nondeterministic finite automaton (with or without \( \epsilon \)-transitions) with at most 8 states that accepts \( L_2 \). Notes: A brief and precise interpretation of the states of your machine is required, and no credit will be given to finite automaton with more than 8 states.
3. [10 points]

(a) [7 points] Let \( L = \{a^ib^j\epsilon^k \mid i, j, k \geq 0, i + k = j\} \). Prove that \( L \) is not regular by using the pumping lemma for regular languages.

(b) [3 points] Using the closure properties of regular languages (regularity-preserving operators) to show that \( L \) is not regular.