Notes:

- Read Course Information: Section 7 (Miscellaneous) and Section 9 (Academic Dishonesty or Misconduct).
- When you are giving a construction, example, etc., provide a justification with your argument. Your solutions to numerical problems must contain the derivation of your answers. In all of your presentations, strive for correctness, completeness, and clarity. When in doubt about the assumptions of problems, the interpretations of wording, etc., consult the instructor.
- You should strive to complete all problems assigned, and a subset of them will be graded.

1. Read the notes above carefully.

2. For each of the following languages, construct a finite automaton (deterministic, nondeterministic, or nondeterministic finite automaton with \( \epsilon \)-transitions — unless unless specifically stated) that accepts the language. Give the key idea(s) for your construction, and brief and precise interpretations for the states of the machine.
   
   (a) \( x \in \{0, 1\}^* \mid \#_0(x) = \#_1(x) \) and every prefix of \( x \) has at most one more 0 than 1s and at most one more 1 than 0s.

   (b) \( a^i b^j \mid i, j \geq 0, \) and \( i + j \) is even.

   (c) \( x \in \{0, 1, 2\}^* \mid \#_1(x) + \#_2(x) \) is divisible by 3.

   (d) \( x \in \{0, 1\}^* \mid \) there exist two 0s in \( x \) that are separated by a string of length 5\( k \) for some \( k \geq 0 \).

   (e) The set of all strings over the alphabet \( \{a, b, c\} \) that yield the same value when evaluated from left to right by “multiplying” according to the following table in Figure 1.

   For examples: \((a \odot b) \odot b = (c \odot b) = a\) and \((a \odot (b \odot b)) = (a \odot a) = a\), whereas \(((a \odot b) \odot c) = (c \odot c) = b\) and \((a \odot (b \odot c)) = (a \odot c) = c\).

   \[
   \begin{array}{c|ccc}
   \circ & a & b & c \\
   \hline
   a & a & c & c \\
   b & b & a & c \\
   c & c & a & b \\
   \end{array}
   \]

   Figure 1: A non-associative multiplication table for \( \circ \).

3. Prove that there does not exist any deterministic finite automaton that accepts the following language:

   \[\{ab^n a^{2n} \mid n \geq 1\}\]

4. More problems may be given in later version.