Notes:

- Read Course Information: Section 7 (Miscellaneous) and Section 9 (Academic Dishonesty or Misconduct).
- When you are giving a construction, example, etc., provide a justification with your argument. Your solutions to numerical problems must contain the derivation of your answers. In all of your presentations, strive for correctness, completeness, and clarity. When in doubt about the assumptions of problems, the interpretations of wording, etc., consult the instructor.
- You should strive to complete all problems assigned, and a subset of them will be graded.

1. Read the notes above carefully.

2. You may need to review the prerequisite materials in discrete mathematics to have sufficient working knowledge, and then do the following exercises.

3. In each case below, find an expression for the indicated set that involves the set-theoretic operators $\cup$, $\cap$, and $\neg$ (complementation):

   (a) $\{ x \mid x \in A \text{ or } x \in B \text{ but not both} \}$.
   (b) $\{ x \mid x \text{ is an element of exactly one of the three sets } A, B, \text{ and } C \}$.
   (c) $\{ x \mid x \text{ is an element of exactly two of the three sets } A, B, \text{ and } C \}$.
   (d) $\{ x \mid x \in A \text{ or } x \in B \text{ or } x \in C \}$.

4. Read/review “binary relation”, “equivalence relation”, “equivalence class”, and “index of an equivalence relation” in a typical discrete mathematics text, and do the following problem.

   Let $P$ denote the set of all compound propositions involving the simple/atomic propositions $p$, $q$, and $r$ and the logical connectives $\lor$, $\land$, and $\neg$ (complementation). (Included in $P$ are the tautology proposition $true$ and the contradiction proposition $false$.) Define a binary relation $R$ on $P$ by:

   $$s \mathrel{R} t \text{ if and only if } s \equiv t,$$

   where $\equiv$ denotes the logical equivalence in propositional logic.

   (a) Show that $R$ is an equivalence relation on $P$.
   (b) How many equivalence classes of $R$ are there? [For every element $p \in P$, the equivalence class (of the equivalence relation $R$ on $P$) containing $p$, denoted by $[p]_R$, is the set $\{ t \in P \mid t \mathrel{R} p \}$ — the set of all elements in $P$ that are related to $p$ under $R$. The index of an equivalence relation is the number of its equivalence classes. ] List some elements in the equivalence class containing the compound proposition $(p \land q) \lor (\neg r)$. List some elements in the equivalence class containing the tautology $true$, and some elements in the equivalence class containing the contradiction $false$.

5. Write a quantified statement that says there are exactly two elements $x$ in the set $A$ for which the proposition $P(x)$ holds.

6. For arbitrary predicate (first-order statements) $A$ and $B$ over an arbitrary common domain, prove, or disprove via a counter-example, each of the following logical equivalences in first-order logic:

   (a) $\forall x (A(x) \lor B(x)) \equiv (\forall x A(x)) \lor (\forall x B(x))$.
   (b) $\forall x (A(x) \land B(x)) \equiv (\forall x A(x)) \land (\forall x B(x))$.
   (c) $\exists x (A(x) \lor B(x)) \equiv (\exists x A(x)) \lor (\exists x B(x))$.
   (d) $\exists x (A(x) \land B(x)) \equiv (\exists x A(x)) \land (\exists x B(x))$. 