1. Let $\Sigma$ be an alphabet. Clearly there exist two distinct strings $x$ and $y$ over $\Sigma$ that satisfy the commutativity: $xy = yx$ (for instance, when $x = \epsilon$).

Is the commutativity possible for non-empty strings $x$ and $y$ (in $\Sigma^+$)? Prove that this cannot happen, or describe/prove precisely the circumstances under which it can.

3. Let $\Sigma$ be the alphabet $\{0, 1\}$. Denote by $L$ the language $\{u \in \Sigma^* \mid u = vv \text{ for some string } v \in \Sigma^*\}$. Prove or disprove that the language $L$ can be expressed as the concatenation of two “non-trivial” languages $L_1$ and $L_2$ over $\Sigma$: $L_1 \neq \{\epsilon\}$ and $L_2 \neq \{\epsilon\}$ and $L = L_1L_2$.

4. Do [Sip12] Chapter 1, exercises 1.5 (h), 1.6 (i), 1.12; and problems 1.33, 1.37, and 1.41.

Note:
(a) For deterministic finite automaton construction, give brief and precise interpretations for the states of the machine, and
(b) For exercise 1.12, ignore the part on regular expression.

5. Do [Sip12] Chapter 1, problems 1.40 (b), 1.59, and 1.61.