7. Let $\Sigma$ be the alphabet $\{a, b\}$. Explain your answers.

(a) $\{w \in \Sigma^* \mid \text{for some } u \in \Sigma^2, w = uu'vu\}$. 
(b) $\{w \in \Sigma^* \mid ww = vvv\}$. 
(c) $\{w \in \Sigma^* \mid \text{for some strings } u \text{ and } v \text{ over } \Sigma, vv = vvuv\}$. 
(d) $\{w \in \Sigma^* \mid \text{for some string } u \text{ over } \Sigma, wvw = uwv\}$. 

8. Let $\Sigma$ be the alphabet $\{0, 1\}$. Denote by $L$ the language $\{u \in \Sigma^* \mid u = vv \text{ for some string } v \in \Sigma^*\}$. Prove or disprove that the language $L$ can be expressed as the concatenation of two “non-trivial” languages $L_1$ and $L_2$ over $\Sigma$: $L_1 \neq \{\epsilon\}$ and $L_2 \neq \{\epsilon\}$ and $L = L_1L_2$.

9. For each of the following languages, construct a deterministic finite automaton that accepts the language. You need to give brief and precise interpretations for the states of the machine.

(a) $L_1 = \{x \in \{0, 1\}^* \mid x \text{ contains three consecutive } 1\text{s}\}$. 
(b) $L_2 = \{x \in \{0, 1\}^* \mid x \text{ does not end with } 11\}$. 

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Notes:

- Read Course Information: Section 7 (Miscellaneous) and Section 9 (Academic Dishonesty or Misconduct).
- When you are giving a construction, example, etc., provide a justification with your argument. Your solutions to numerical problems must contain the derivation of your answers. In all of your presentations, strive for correctness, completeness, and clarity. When in doubt about the assumptions of problems, the interpretations of wording, etc., consult the instructor.
- You should strive to complete all problems assigned, and a subset of them will be graded.
9. Let $L \subseteq \{0, 1\}^*$ be the language of all strings such that there exist two 0s separated by a number of positions that is a non-zero multiple of 5. For example, the string 1001110 is not in $L$, but the string 10110110111110 is in $L$. Intuitively, any deterministic finite automaton accepting $L$ must “remember” 5 positions in order to determine the membership of the string — this is a general idea but not a proof.

Prove that every deterministic finite automaton accepting $L$ must have at least $2^5$ states.